High-order Wavelets for Hierarchical Refinement in Inverse Rendering

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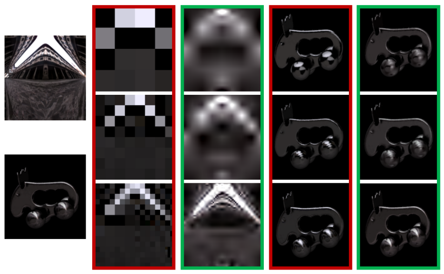

Figure 1: *Iterative refinement of illumination by adding detail coefficients based on a splitting criterium. Reconstructions for both Haar (red) and the smoother Coiflet (green) wavelet bases are shown. Haar has a tendency to introduce disturbing high frequencies around edges.*

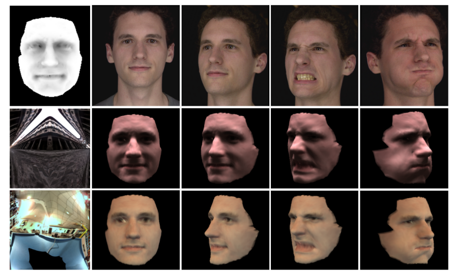

Figure 2: *Reconstruction of a temporal face dataset under different lighting conditions and estimated with the hierarchical refinement method. Ray traced occlusion maps, BRDF slices and lighting environment map are combined in the triple product integral calculation.*

1 Introduction

It is common to use factored representation of visibility, lighting and BRDF in inverse rendering. Current techniques use Haar wavelets to calculate these triple product integrals efficiently [Ng et al. 2004]. Haar wavelets are an ideal basis for the piecewise constant visibility function, but suboptimal for the smoother lighting and material functions. How can we leverage compact high-order wavelet bases to improve efficiency, memory consumption and accuracy of an inverse rendering algorithm? If triple product integrals can be efficiently calculated for higher-order wavelets, the reduction in coefficients will reduce the number of calculations, therefore improving performance and memory usage. Some BRDFs can be stored five times more compactly.

Current inverse rendering algorithms rely on solving large systems of bilinear equations [Haber et al. 2009]. We propose a hierarchical refinement algorithm that exploits the tree structure of the wavelet basis. By only splitting at interesting nodes in the hierarchy, large portions of less important coefficients can be skipped. The key of this algorithm is only splitting nodes of the wavelet tree that contribute to the solution of the system M (see Algorithm 1). It is critical to use high-order wavelets for this, as Haar wavelets can only introduce high frequencies which lead to blockiness.

2 Our Apprach

In forward rendering, the rendering equation can be expressed as a triple product integral of the visibility V_i, environment map $\tilde{L}_j$ and BRDF ρ_k, all three represented in the wavelet domain:

$$B(x,\omega_0) = \sum_i \sum_j \sum_k L_i V_j \rho_k \int_\Omega \Psi_i(\omega)\Psi_j(\omega)\Psi_k(\omega) d\omega \quad (1)$$

In contrast to the regular Haar solution of Ng. [Ng et al. 2004], higher-order wavelets have overlapping support, resulting in more complex binding coefficients $\int_\Omega \Psi_i(\omega)\Psi_j(\omega)\Psi_k(\omega)d\omega$. This calculation is much more complex and needs to be precalculated. Our method exploits the hierarchical nature and vanishing moments of wavelets, to efficiently solve this sparse tensor.

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Instead of solving for all coefficients of the wavelet tree of the environment map, we can also use the hierarchical nature of wavelets to iteratively refine the estimation. The idea is to start with a smaller wavelet tree and adaptively add more detail coefficients by splitting the leaf nodes. Instead of previous methods [Peers and Dutré 2005], where they use properties of the coefficients, we use the rank of the system as a splitting criterium, while also allowing a mix of high-order wavelet bases to improve convergence (see Algorithm 1). Figure 1 gives a comparison of the hierarchical method for both a piecewise constant Haar Basis and a smoother Coiflet basis. Reconstruction of a temporal dataset with this hierarchical method is shown in Figure 2.

Algorithm 1 Hierarchical Refinement Scheme

```
M: initialize system to solve at root node of wavelet tree
repeat
    K: set of possible nodes for refinement
    for k ∈ K do
        concatenate k to M: M = M|M_k
        if rank(M) ≠ full then remove k from K
        end if
    end for
    Solve M for L̃
until no splits left
```

References

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